Retrofocusing techniques in a waveguide for acoustic communications (L)

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In the paper “Retrofocusing technique for high rate acoustic communications” [M. Stojanovic, J. Acoust. Soc. Am. 117, 1173–1185 (2005)], it is suggested that the time reversal approach requires a large number of array elements to compete with other approaches. Here the analysis of that paper is extended with a modified example to compare the performance of various approaches in three respects: (1) array element distribution across the water column, (2) channel normalization, and (3) phase delay across the array. In contrast, our results show that the time reversal approach combined with channel equalization can offer nearly optimal performance with a very small four element array. © 2007 Acoustical Society of America. [DOI: 10.1121/1.2721877]

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A recent paper by Stojanovic\textsuperscript{1} explored various possible approaches to the acoustic communication between a single element and a multi-element array including time reversal\textsuperscript{2,3} and channel equalization,\textsuperscript{4} under the assumption that the channel transfer function is available to the transmitter and/or the receiver. Motivated by the limitations of the time reversal approach (SNR\textsubscript{0}) due to residual intersymbol interference (ISI),\textsuperscript{5,6} optimization techniques (SNR\textsubscript{1} and SNR\textsubscript{2}) were developed which simultaneously eliminate ISI and maximize the output signal-to-noise ratio (SNR), while maintaining maximal data rate in a given bandwidth subject to a constraint on the transmitted symbol energy. In addition, an optimal solution involving channel equalization (SNR\textsubscript{4}) was proposed to offer the best performance at the expense of the complexity of the overall system. The paper\textsuperscript{1} provides theoretical bounds on the performance of various approaches in terms of the output SNR and examines the performance using a simple model channel.

However, the simple channel model chosen for the analysis in Ref. 1 did not include important propagation physics that, if included, potentially alter some of the conclusions in Ref. 1. In this letter, the analysis of the paper is extended to arrays that are capable of utilizing the spatial diversity of the model channel to compare the performance of the various approaches. It is found that the various approaches collapse to basically four different approaches: (1) time reversal alone, (2) time reversal with equalization, (3) equalization with a fixed transmit array, and (4) optimal approach. Approach (3) does not use the channel information and generally performs poorly as compared to the other approaches except approach (1). In this letter, we address the following three components in the model channel used for performance analysis: (i) array element spacing, (ii) channel normalization, and (iii) phase delay across the array.

(i) The major issue is the inter-element spacing employed in the channel model. The typical requirement of the element spacing being equal to or less than half the wavelength ($d<\lambda/2$) to avoid spatial aliasing can be relaxed for a vertical array in an underwater waveguide since the propagation angles are almost horizontal (broadside to the array) as extensively studied in matched field processing.\textsuperscript{7} Thus, the important question is how to distribute array elements across the water column given the number of elements ($M$) where the element spacing ($d$) constrains the aperture of the array [i.e., $L=d(M-1)$]. Specifically, the example in the paper using $M=4$ elements with $d=\lambda/2$ spacing has an aperture of $L=3\lambda/2$ which is not large enough to resolve the three multipaths propagating at low grazing angles (e.g., $\theta =0^{\circ}, 2.7^{\circ}, 5.4^{\circ}$). Then a four-element array with this small inter-element spacing behaves effectively as a clustered single element. As a result, there needed to be an increase in the number of elements until the time reversal array reached an aperture sufficient to resolve the three multipaths, requiring a large number of array elements (e.g., $M=32$) to compete with the other approaches.

Here we use the same number of array elements, $M=4$, but with an element spacing of $d=4\lambda$ which provides a suf-

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sufficient aperture to benefit from the spatial diversity of the field as illustrated in Fig. 1. The difference between \( \lambda/2 \) and 4\( \lambda \) element spacing results in as many as eight times the number of elements. The composite channel function \( \gamma(t) \) is shown in Fig. 1(a) (top) while the impulse response of the overall system with time reversal \( q(t) \) is shown in Fig. 1(a) (bottom). The performance of time reversal communications depends on the behavior of \( q \)-function whose Fourier transform is \( \gamma(t) \) defined in Eq. 12 of Ref. 1 as the summation of autocorrelation functions of each channel impulse responses \( C_i(f) \). To minimize the ISI, it would be desirable to have a \( q \)-function that approaches a Dirac delta function. Even with \( M=4 \) elements, however, the \( \gamma(t) \) flattens out and the overall system obtained with time reversal \( q(t) \) shows much smaller sidelobes as compared to those in Fig. 3 of Ref. 1 with \( d=\lambda/2=0.05 \) m. In fact, the aperture of \( M=4 \) elements with 4\( \lambda \) spacing is still modest with \( L=1.2 \) m in 75-m deep water.

Figure 1(b) summarizes the corresponding performance results using \( M=4, L=1.2 \) m array. Several observations can be made. First, Fig. 1(b) looks very similar to Fig. 4 of Ref. 1 where \( L=1.6 \) m with \( M=32 \) and \( d=\lambda/2 \), clearly indicating the redundancy of the array elements with smaller element spacing. Second, note that there are only four distinct curves shown in Fig. 1(b) while there are seven different legends. Apparently, all the optimal techniques (SNR\(_1\), SNR\(_2\), SNR\(_4\)) as well as passive time reversal with equalization (SNR\(_{\text{opt}}\)) collapse onto one another, which can be grouped as an optimal approach (4). The other three curves represent: time reversal alone [SNR\(_{\text{opt}}\), dashed with squares, approach (1)], active time reversal with equalization [SNR\(_{\text{opt}}\), cross, approach (2)], and equalization with a fixed transmit array [SNR\(_{\text{opt}}\), circle/dashed, approach (3)]. Third, approach (2) also offers performance very close to the optimal approach (4), suggesting that time reversal alone (either active or passive) can be optimized in conjunction with channel equalization by removing the residual ISI. Approach (4) shows a linear curve proportional to the input SNR, \( E/N_0 \), which provides an upper bound to the other approaches. Fourth, it is noticeable that time reversal approach (1) saturates eventually due to residual ISI at a value \( 1/\rho \) (defined in Eq. 34 of Ref. 1) which is determined by the channel function \( \gamma(t) \). On the other hand, approach (1) is almost identical to the optimal approach (4) at low SNR below \( E/N_0=7 \) dB. This is because the impact of ISI through \( \rho \) will be either comparable to or negligible with respect to the noise level at low SNR and as \( E/N_0 \rightarrow 0 \), the output SNR\(_0\) reduces to \( \text{SNR}_{\text{opt}} = (E/N_0) \int_0^\infty X(f) \gamma(f) df \), linearly proportional to \( E/N_0 \). The characteristics of the \( q \)-function affect the point of separation from the optimal curves. Finally, approach (3), which does not use the channel knowledge consistently, is outperformed (about 2 dB) by the optimal approach (4).

(ii) For convenience, the \( q \)-function is normalized such that \( q(0)=1 \) in Fig. 1 where \( q(0) \) represents the total energy of the channel impulse responses (three multipaths in our example). However, this normalization should not be imposed when calculating the performance bounds since we have a constraint on the transmitted symbol energy \( E \) independent of \( M \). This allows us to distinguish the impact of varying the number of array elements from the impact of varying the total transmitted power.\(^6\) For uplink scenarios, one should pick up more energy by increasing the number of receivers for a fixed transmitted power.

Performance sensitivity with respect to the number of array elements (or, equivalently, array size) is shown in Fig. 2(a) illustrating the output SNR as a function of \( M \) for a given symbol SNR of \( E/N_0=20 \) dB. The performance characteristics are quite different from Fig. 5 of Ref. 1 where \( d=\lambda/2 \) with the normalization imposed. Three observations can be made. As in Fig. 1(b), the performance of all of the optimal focusing techniques collapse onto each other as well as approach (2) forming a single curve, while the perfor-
The least common denominator was frequency-dependent such that phenomenon normally would not occur if the phase delay oscillatory characteristics.

where the spatial diversity is exploited maximally.

Second, approach (3) is outperformed by the optimal approach (4) as in Fig. 1(b). In addition, approach (3) initially shows better performance than approach (1) up to $M=4$, but is saturated and eventually outperformed by approach (1) due to an inefficient use of energy without exploiting the channel knowledge. Finally, the oscillatory behavior of time reversal approach is quite peculiar with a period of $M=5$, corresponding to about 2 m in the array aperture. The period of 2 m turns out consistent with the period shown in Fig. 6 of Ref. 1, which is explained below.

(iii) The vertical wave number interference between two multipaths denoted by $\Delta_{ij}$ is

$$
\Delta_{ij} = \frac{2\pi}{|k_i - k_j|} = \frac{\lambda}{|\sin \theta_i - \sin \theta_j|},
$$

where $k_i$ is a vertical wave number. For the three multipaths shown in Fig. 1 of Ref. 1, the interference distances at 15 kHz are $\Delta_{13} = 1.06$ m, $\Delta_{23} = 2.13$ m, and $\Delta_{12} = 2.12$ m, respectively. The least common denominator (LCD) is then $\Delta = 2.12$ m, which is approximately the period of the oscillatory characteristics.

Since we are dealing with a broadband signal of 5 kHz bandwidth centered at 15 kHz in the example, the oscillatory phenomenon normally would not occur if the phase delay was frequency-dependent such that $\phi(f) = 2\pi(d/\lambda)\sin \theta$ where $\lambda = c/f$. This is because the interference varies with frequency [or $\lambda$ in Eq. (1)] and smears out over the band of frequency. It should be mentioned that the frequency dependent phase across the array is a direct result of the simple time delay relationship between the array elements. Figure 2(b) re-evaluates Fig. 2(a) using the frequency-dependent phase $\phi(f)$. As expected, the oscillatory behavior of time reversal approach (1) clearly has diminished.

Erratum: Retrofocusing techniques in a waveguide for acoustic communications (L) [J. Acoust. Soc. Am. 121(6), 3277–3279 (2007)]

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In the subject paper, the curves in Fig. 2 that are labeled as “Approach 3” are incorrect. A suitably revised figure (now labeled Fig. 1) with the correct curves is given here.

FIG. 1. Performance of various techniques on the example channel: output SNR versus M for fixed symbol SNR = E/N₀ = 20 dB. (a) d=4λ without the channel normalization q(θ) = 1 imposed. Note the aperture of M=35 elements array is 17 m in 75-m-deep water. (b) Reproduction of plot (a) when the phase delay is frequency dependent such that φ(f) = 2π(f/dλ) sin θ.

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