Time reversed reverberation focusing in a waveguide

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Time reversal mirrors have been applied to focus energy at probe source locations and point scatterers in inhomogeneous media. In this paper, we investigate the application of a time reversal mirror to rough interface reverberation processing in a waveguide. The method is based on the decomposition of the time reversal operator which is computed from the transfer matrix measured on a source-receiver array \cite{Prada et al., J. Acoust. Soc. Am. 99, 2067–2076 (1996)}. In a similar manner, reverberation data collected on a source-receiver array can be filtered through an appropriate temporal window to form a time reversal operator. The most energetic eigenvector of the time reversal operator focuses along the interface at the range corresponding to the filter delay. It is also shown that improved signal-to-noise ratio measurement of the time reversal operator can be obtained by ensonifying the water column with a set of orthogonal array beams. Since these methods do not depend upon \textit{a priori} environmental information, they are applicable to complex shallow water environments. Numerical simulations with a Pekeris waveguide demonstrate this method. © 2002 Acoustical Society of America. \ DOI: 10.1121/1.1479148

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I. INTRODUCTION

Recent experiments have demonstrated time reversal mirrors (TRMs) in the ultrasonic laboratory\textsuperscript{1,2} and in ocean acoustics.\textsuperscript{3–5} Time reversal mirrors often involve a probe source or scattering from compact objects. In this paper we examine the application of a TRM to scattering from a rough boundary in a waveguide. The method employed is based upon the DORT method for the decomposition of the time reversal operator (TRO).\textsuperscript{6–8} In this case, probe pulses are projected from the source-receiver array (SRA) and the backscattered signal due to the interface roughness is recorded. A temporal window corresponding to a desired focusing range from the SRA is applied to the reverberation data and the resulting time series data are used to form the TRO. The highest energy eigenvector of the TRO corresponds to the SRA weighting function for focusing along the interface at the desired range. This extends the previous work on time reversed focusing in two ways. First, reverberation from the rough interface which is often regarded as a source of clutter in sonar performance is utilized to probe the waveguide. Second, an extended interface as opposed to a compact scatterer is used to form the time reversal operator. In Sec. II we describe the scattering model implemented to simulate reverberation from a rough interface in a Pekeris waveguide. Section III describes the processing of the reverberation data using time reversal techniques. A generalized method utilizing orthogonal array beams for measuring the time reversal operator is also described. In Sec. IV simulations in a Pekeris waveguide illustrate these methods.

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II. REVERBERATION MODEL

We consider a geometry where a source-receiver array is oriented vertically in the water column as shown in Fig. 1. Roughness at the sediment interface scatters an outgoing wave generated at the SRA and the reverberation is recorded at each element of the array. For the simulations in this paper, the backscattered pressure field is computed from perturbation theory. A Pekeris waveguide with a flat pressure release surface at \(z=0\) and perturbed interface at \(z=D + \gamma(r)\) is considered where \(\gamma\) is the deformation from a flat surface and \(r\) is the horizontal position vector; position vectors in three dimensions are denoted by \(\mathbf{x}=(r,z)\). Factoring out a harmonic time dependence \(\exp(-i\omega t)\) from the wave equation for pressure yields

\[
\nabla^2 p + k^2 p = \delta(\mathbf{x} - \mathbf{x}_s), \tag{1}
\]

where the subscripts \(j=1,2\) will denote material parameters above and below the sediment interface, respectively, and the source location is \(\mathbf{x}_s\). The total pressure is expanded in a perturbation series

\[
p = p_0 + s, \tag{2}
\]

where \(p_0\) is the outgoing pressure field of the unperturbed problem (flat bottom) and \(s\) is the leading order scattered pressure due to interface roughness. The scattered pressure \(s\) satisfies Eq. (1) without the explicit source term; it is forced through the interface conditions. An integral equation for \(s\) follows from applying Green’s theorem,
FIG. 1. Pekeris waveguide with a rough interface. For simulations in Sec. IV, $c_1 = 1500 \text{ m/s}$, $c_2 = 1600 \text{ m/s}$, $\rho_1 = 1000 \text{ kg/m}^3$, $\rho_2 = 1900 \text{ kg/m}^3$, and $D = 100 \text{ m}$. The rough interface is modeled with one-dimensional Goff–Jordan power-law spectrum (Ref. 18) with a correlation length of $L = 15 \text{ m}$ and mean square roughness $(\gamma') = 0.01 \text{ m}^2$.

\[ s(r,z) = \rho_1 \int dr' \left[ \frac{1}{\rho} \frac{\partial G(r-r',z,D)}{\partial z'} \right] G(r-r',z,D) \right], \quad (3) \]

where $G$ is the Green's function of the unperturbed problem, the contour of integration is along $z = D$, and the square brackets denote the jump of the enclosed quantity across the interface. The jump conditions are obtained by expanding the interface conditions for pressure to leading order about $z = D$ yielding

\[ [s(r,D)] = -\gamma(r) \frac{\partial p_0(r,D)}{\partial z}, \quad (4) \]

\[ \left[ \frac{1}{\rho} \frac{\partial s(r,D)}{\partial z'} \right] = -\gamma(r) \left[ \frac{1}{\rho} \frac{\partial^2 p_0(r,D)}{\partial z'^2} \right] + \nabla_y \gamma \left[ \frac{1}{\rho} \nabla_z p_0(r,D) \right], \quad (5) \]

where $\nabla_y$ is the horizontal gradient. The perturbation theory assumes a small ratio of roughness height to acoustic wavelength $k \gamma \ll 1$, small interface slope $|\nabla_y| \ll 1$, and that multiple scattering can be neglected. The modal representation of the scattered field is derived by expanding the Green's function and $p_0$ in terms of modes. For computational efficiency we consider two-dimensional propagation with a line source geometry where the far-field Green's function is

\[ G(x,x') = -i \sum_m \phi_m(z) \phi_m(z') \frac{e^{ih_m|x-x'|}}{h_m}, \quad (6) \]

and $\{ \phi_m, h_m \}$ are the set of propagating modes and horizontal wave numbers of the unperturbed problem. Substituting Eqs. (4)–(6) into Eq. (3) and evaluating the expression for the backscattered field at $x = 0$ yields

\[ s(z;z_f) = \frac{1}{\rho} \sum_{m,n} \left( h_m h_n + \frac{k^2}{\rho} \right) \phi_m(D) \phi_n(z_f) \]

\[ \times \phi_n(D) \phi_m(z_f) \int_{-\infty}^{\infty} G(x) e^{i(h_m h_n + k^2/\rho)|x|} dx. \quad (7) \]

Equation (7) has been simplified using the boundary condition relating the modes functions of the Pekeris waveguide and their derivatives at the interface

\[ \frac{1}{\rho} \frac{d \phi_m(D)}{dz} = \frac{1}{\rho_2} \sqrt{k_m^2 - k_2^2} \phi_m(D). \quad (8) \]

The backscattered field due to multiple sources is obtained by superposition and broadband pulses are modeled with Fourier synthesis.

III. TIME REVERSAL PROCESSING OF REVERBERATION DATA

This section describes our method for processing reverberation data collected on the source-receiver array. The data are recorded on an $N$ channel SRA which is configured to transmit an incident pulse of length $t$ starting at $t = t_0$ (transmit mode). The resulting reverberation time series is then recorded on each element (receive mode). The switching time between transmit and receive modes is assumed to be small so that only the short range reverberation signal is lost. Assuming that only single scattering is important, a specific range $R$ will contribute to the received reverberation over a time window

\[ w(t;R,\Delta) = \begin{cases} 1, & |t - t_c - t_0| \leq \Delta/2 \\ 0, & |t - t_c - t_0| > \Delta/2. \end{cases} \quad (9) \]

where $t_c = 2Rc_0^{-1}$ is the approximate round-trip travel time and $\Delta$ is the width of the temporal window which depends on the pulse length and the dispersive properties of the waveguide; for the examples in this paper we choose $\Delta = \tau$. The reference speed $c_0$ is an average modal group speed. Due to dispersion in the waveguide, a span of ranges of width $\Delta r$ (which depends on the time window $\Delta$) will contribute to the reverberation in the windowed time series. We apply the DORT method to the central frequency component of the reverberation data to obtain a SRA weighting vector which focuses back to the interface at the range $R$. The weighting vector used is the highest energy eigenvector of the time reversal operator. In the following the DORT method for point scatterers is summarized and a generalization of the method to obtain higher signal-to-noise measurements of the eigenvectors is described.

**Time reversal operator**

The time reversal invariance of the wave equation implies that in principle energy can be focused back to its source. Direct experimental verification of time reversal focusing in the oceanic waveguide has been recently demonstrated using a source-receiver array and a probe source. Time reversal mirrors have also been applied to focusing on
scatterers in inhomogeneous media.\textsuperscript{1,2,6,7,15} In the case of several point targets whose scattered waves overlap in the time domain, decomposition of the time reversal operator by the DORT method yields SRA weighting functions for focusing at each scatterer separately. Importantly, this method assumes no \textit{a priori} knowledge of the waveguide properties. Previous implementations of the DORT method were based on measurements of the element-to-element impulse responses of the source-receiver array. Here, an equivalent method based on orthogonal array beams is described for measuring the time reversal operator. Array beams enhance the backscattered signal strength by ensonifying the water column with the full source array rather than single elements as described in the original DORT formulation.

We consider a source-receiver array with \( N \) elements. The original DORT algorithm is based on the measurement of the response matrix \( k_{ij}(t) \), the backscattered field on channel \( i \) of the SRA due to an impulsive signal broadcast from channel \( j \). The Fourier transform of this quantity is the transfer matrix \( K_{ij}(\omega) \). We assume that the fluid density is uniform over the SRA so that \( K \) is a symmetric matrix by reciprocity. The time reversal operator is defined by \( K^*K \) where the asterisk denotes complex conjugation; it is a Hermitian matrix with \( N \) orthogonal eigenvectors and non-negative eigenvalues.\textsuperscript{6} The eigenvectors correspond to the SRA weighting functions that focus on individual scatterers in the waveguide. In the ocean environment, measurement of the response matrix \( k_{ij}(t) \) can be difficult due to significant background noise levels particularly for the weaker returns associated with long ranges. Greater signal-to-noise ratios are possible by probing the waveguide with \( N \) orthogonal beams rather than the individual source array elements. Given a complete set of orthogonal array weighting vectors \( S = \{ e_j \mid 1 \leq j \leq N \} \), the beam response \( \tilde{k}_{ij}(t) \) is the backscattered field projected onto the \( e_j \) direction due to an array impulse weighted by the source function \( e_i \). The Fourier transform of \( \tilde{k}_{ij} \) is \( \tilde{K} \) and related to \( K \) by a similarity transformation,

\[
\tilde{K} = E^TKE,
\]

where the superscript \( T \) denotes transpose and \( E \) is an orthogonal matrix of the column vectors \( S \) satisfying \( E^TE = EE^T = I \). The original DORT method is recovered when \( E = I \). We note that \( \tilde{K} \) is also a symmetric matrix so that reciprocity also applies in array beam space. The time reversal operator in this representation satisfies \( \tilde{K}^* \tilde{K} = E^T \tilde{K}^* \tilde{K} E \) which can be decomposed in terms of the eigenvalues \( \lambda_i \) and eigenvectors \( u_i \) of \( \tilde{K}^* \tilde{K} \) as

\[
\tilde{K}^* \tilde{K} = \sum_{i=1}^{N} \lambda_i (E^T u_i)(E^T u_i)^H,
\]

where the superscript \( H \) denotes the conjugate transpose. Thus the time reversal operator in array beam space decomposes into the same eigenvalues \( \lambda_i \) with eigenvectors \( E^T u_i \). If the number of array elements is a power-of-two, a convenient set of array beams which maximize the transmitted power are the Hadamard–Walsh functions defined by the recursion relationship,\textsuperscript{16}

\[
H_N = \begin{bmatrix} H_{N/2} & H_{N/2} \\ H_{N/2} & -H_{N/2} \end{bmatrix}, \quad H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.
\]

The matrix \( E \) of normalized weighting functions is then \( E = (1/\sqrt{N})H_N \).

It is appropriate to mention here that we should be able to achieve TRM focusing from a single reverberation return (or single snapshot) instead of the \( N \) reverberations required to construct a TRO since a segment of reverberation data is considered as a transfer function from the distribution of sources along the interface. A single snapshot, however, requires a high signal to noise ratio (SNR) whereas the TRO

FIG. 2. (a) Eigenvalues and (b) amplitude of the first eigenvector of the time reversal operator. The time reversal operator is constructed from the reverberation returns at the center frequency of 200 Hz after applying a time window of \( \Delta = 50 \) ms at \( t_0 = 1.58 \) s which corresponds approximately to a round-trip travel time from the array to 1 km range. The vertical source/receive array (SRA) consists of 32 elements with the top element at 2 m depth. The source function is a Gaussian impulse with a width of \( \tau = 50 \) ms resulting in a patch size of \( c_\tau r/2 = 37.5 \) m. In comparison, the magnitude of the transfer function at the center frequency due to a point scatterer located at \((x,z) = (1000,100)\) m is superposed (dashed). The overlap of the two vectors is 0.9.
exploits a redundancy in the $N$ measurements such that it effectively increases the SNR by $20 \log N$ dB above the single snapshot measurement. The other issue is that a single snapshot, either in a phone or beam space, can hardly ensonify the field uniformly along the interface as compared to the $N$ measurements in TRO approach. Finally, the preceding approach assumes that the rough interface and the environment are frozen during the $N$ snapshot measurement. In practice, reverberation from all azimuthal directions will contribute to the field measured on the array and the environment will fluctuate. The azimuthal contributions to the reverberation can be modeled with random realizations of the rough surface. We observe that a single snapshot sample shows similar focusing as the $N$ measurements in TRO approach in Sec. IV, but more detailed analysis is in preparation to include the effect of random realizations of the roughness.

**IV. SIMULATIONS**

Simulations in a Pekeris waveguide are used to demonstrate time reversed reverberation focusing in this section. The waveguide parameters are $\rho_1 = 1000 \text{ kg/m}^3$, $c_1 = 1500 \text{ m/s}$, $\rho_2 = 1900 \text{ kg/m}^3$, $c_2 = 1600 \text{ m/s}$, bottom attenuation $\alpha_2 = 0.8 \text{ dB/} \lambda$, and depth $D = 100 \text{ m}$ for the waveguide shown in Fig. 1. The surface is pressure release and the rough interface realizations are modeled with the one-dimensional Goff–Jordan power-law spectrum

$$P(q) = \pi L \left(1 + (qL)^2\right)^{-3/2},$$

where $q$ is the horizontal wave number and $L = 15 \text{ m}$ is the correlation length of the roughness. In the simulations, the rough surface realizations extend from $0 \leq x \leq 3 \text{ km}$ with a root mean square roughness height of 10 cm. The source-receiver array is vertical and consists of 32 transducer elements located in range at $x = 0 \text{ m}$ and in depth at $z_i = \begin{cases} 0, \text{ for } i = 1, & 32 \text{ meters.} 
\end{cases}$ The source function used to probe the reverberation is a Gaussian impulse with amplitude $A$, center frequency $\omega_0$, width $\tau$, and delay $t_0$ given by

$$s(t) = \frac{A}{\tau} \exp(-\pi^2(t-t_0)^2-i\omega_0 t).$$

The mode functions and eigenvalues used to evaluate the backscattered field in Eq. (7) are computed with KRAKEN.19

FIG. 3. (a) Reverberation from broadside transmission of a 50 ms Gaussian pulse with 32 channels. (b) Reverberation from 50 ms Gaussian pulse with array elements weighted by the first eigenvector of the time reversal operator as shown in Fig. 2(b). (c) and (d) The signal energy summed over all elements corresponding to (a) and (b), respectively. Note that the reverberation signal is maximized at $t = 1.5 \text{ s}$ corresponding to a distance of 1 km from the SRA, approximately 10 dB above the background.
The pulse parameters in our simulations are $t_0 = 50$ ms, $f_0 = (2\pi)^{-1}\omega_0 = 200$ Hz and $t_0 = 0.2$ s. The time reversal operator is computed at the center frequency $f_0$ after applying a time window $w_1 = w(t, 1 \text{ km}, 50 \text{ ms})$ to the reverberation which is modeled by the modal sum of Eq. (7). The eigenvalues and the first eigenvector $e_1$ of the time reversal operator are shown in Fig. 2. For comparison, the transfer function $e_{ps}$ at the center frequency due to a point scatter located at $x = (1000, 100)$ m is superimposed in a dashed line. The overlap between the two vectors is $|e_1 \cdot e_{ps}| \approx 0.9$, a degradation of 1 dB. Figure 3 is a comparison of the reverberation time series due to (a) a broadside transmission from the SRA and (b) a transmission with elements weighted by the first eigenvector of the time reversal operator as shown in Fig. 2(b). The reverberation energy summed over all elements $E(t) = \sum p_i(t)$ of the SRA is shown in (c) and (d), respectively. In Fig. 3(d) the reverberation energy is maximized at $t = 1.5$ s corresponding to a distance of 1 km from the SRA and the amplitude is approximately 10 dB above the background. Figure 4 shows the intensity plot in range and depth at the center frequency of 200 Hz confirming the spatial focusing property. The top panel is due to transmission of the first eigenvector of time reversal operator with the roughness shown in the middle. In comparison, the bottom panel shows the time reversal focusing for a point scatterer at $(x, z) = (1000, 100)$ m. The average modal group speed was assumed 1450 m/s for computation of time window corresponding to 1 km range. Although it is not shown here, time reversal of a single snapshot of the reverberation due to a broadside transmission shows a similar result to Fig. 4 as noted in Sec. III. Figure 5 shows the time-depth distribution of energy at various ranges $x = 0.6, 0.8, 1.0, 1.2$ km from the SRA. As expected, a strong focus of the energy is observed near the bottom at a range of 1 km from the SRA.

V. CONCLUSIONS

Focusing with a time reversal operator is extended to the case of stochastic reverberation returns from a rough sediment interface in an ocean waveguide. The weighting function for the source array is computed from the first eigenvector.
tor of the time reversal operator after selecting a time window corresponding to the intended focal range along the interface. In addition, the time reversal operator, originally formulated in the phone space, is generalized to the array beam space. Array beams will be useful in practical applications because they provide a greater power input than single source elements for measurement of backscatter from the rough interface which are usually very weak. Numerical simulations with a Pekeris waveguide demonstrate the proposed method.

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